

Written Exam for the M.Sc. in Economics, Winter 2018-19

Advanced Macroeconomics: Structural Vector Autoregressive Analysis

About the exam

The topic for this project examination is the Purchasing Power Parity theorem stating that the foreign exchange rate adjusts to changes in relative prices in two countries. Purchasing Power Parity (PPP) states that

$$S_{12} = \frac{P_1}{P_2} \quad (1)$$

where S_{12} is the nominal exchange rate between countries 1 and 2, P_1 is the price level in country 1 and P_2 is the price level in country 2. The real exchange rate is defined as

$$\frac{P_1}{P_2 S_{12}} \quad (2)$$

implying that the expected real exchange rate is equal to unity. If we take the log of the PPP relation in equation (1) and rearrange we obtain

$$\ln P_1 - \ln P_2 - \ln S_{12} = 0. \quad (3)$$

If PPP holds and prices and exchange rates are $I(1)$ -processes, then the three variables must be cointegrated with cointegration vector

$$[1 \quad -1 \quad -1] \quad (4)$$

To avoid that some data sets are more difficult to handle than others, the data sets are artificial (simulated from a known data generating process), and they behave, as close as possible, like actual data. The data is generated by the following Vector Error Correction (VEC) model

$$\Delta y_t = \nu + \alpha \beta' y_{t-1} + \Gamma_1 \Delta y_{t-1} + u_t$$

where

$$y_t = \begin{bmatrix} \ln P_1 \\ \ln P_2 \\ \ln S_{12} \end{bmatrix}, \quad \alpha = \begin{bmatrix} 0.009 \\ 0.014 \\ 0.060 \end{bmatrix}, \quad \beta = [1 \quad -1 \quad -1.5], \quad \nu = \begin{bmatrix} 0.012 \\ 0.001 \\ 0 \end{bmatrix}$$

and $u_t \sim \mathcal{N}(0, \Sigma)$. The remaining parameters are calibrated such that the resulting time series vector more or less behave as actual time series. There are no structural breaks in the data. In addition, the time series vector is generated such that the lag length obtained using any of the three information criteria (Akaike, Schwarz and Hannan-Quinn) suggest 2 lags in the underlying Vector Autoregressive (VAR) model and that the Johansen trace test correctly suggests the presence of 1 cointegration vector in the model. A formal test whether the

PPP relation is present in the cointegration space (under the assumption that there is 1 cointegration vector in the system and that the lag length is equal to 2) can produce both a rejection and a non-rejection depending on the data set used. The data covers the sample 1974M1-2017M12.

It is not possible to detect the true data generating process but it is required that sound arguments are used when discussing the model specification and that choices made are convincingly motivated.

The proposed solution below is based on the dataset 101.mat

The Vector Error Correction Model

Suppose that all three variables in the PPP relation are either $I(1)$ or $I(0)$ and that the underlying data generating process is a 3-dimensional Vector Autoregressive (VAR) model,

$$y_t = \nu + A_1 y_{t-1} + \dots + A_p y_{t-p} + u_t \quad (5)$$

where $y_t = [\ln P_1 \quad \ln P_2 \quad \ln S_{12}]'$, p is the lag length, ν is a constant term and u_t is a 3-dimensional zero mean white noise process with covariance matrix Σ_u such that $u_t \sim (0, \Sigma_u)$. Then we can rewrite the VAR model as the following Vector Error Correction (VEC) model

$$\Delta y_t = \nu + \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + \dots + \Gamma_{p-1} \Delta y_{t-p+1} + u_t \quad (6)$$

where

$$\Pi = -(I_3 - A_1 - \dots - A_p)$$

and

$$\Gamma_i = -(A_{i+1} + \dots + A_p) \quad \text{for } i = 1, \dots, p-1.$$

The rank of Π is equal to the number of cointegration vectors r and can be decomposed as a product of two $3 \times r$ matrices of full rank, $\Pi = \alpha\beta'$ where α is the $3 \times r$ adjustment coefficients and β is the $3 \times r$ cointegration vectors.

1. The data is already in natural logarithms. Construct the real exchange rate and plot the data including the relative price ($\ln P_1 - \ln P_2$) and perform graphical analysis in order to assess the degree of integration of all five variables (the nominal exchange rate, the two price levels, the relative price and the real exchange rate).

Answer: The graphs below show the data and the real exchange rate.

According to the graphs, we clearly see that the price levels are rising over time whereas there is no apparent trend in the nominal exchange rate, see Figure 1. The real exchange rate may appear stationary around a constant mean (this hypothesis will be tested below) thus suggesting that the data may be consistent with PPP, see Figure 2. The mean of the real exchange rate is not equal to zero as implied by PPP. This could reflect transaction costs and other possible barriers.

Figure 1: Nominal exchange rate, domestic price level and foreign price level, 1974M1-2017M12.

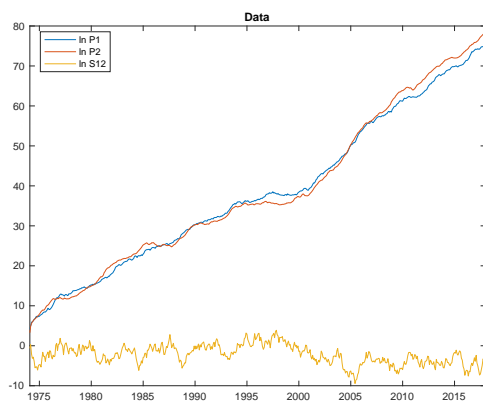
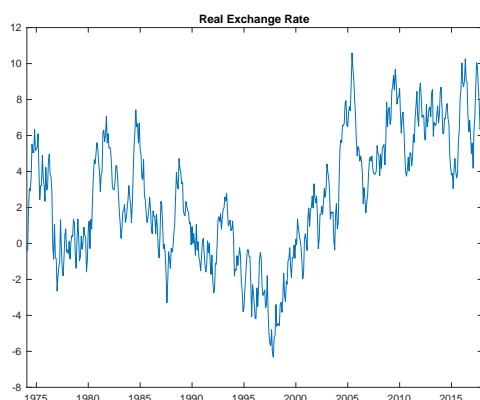


Figure 2: The real exchange rate, 1974:M1-2017M12.



2. Formulate a well-specified VEC model for y_t similar to the VEC model above. Explain your workflow and how you argue for your choice of the number of autoregressive lags.

Answer: The first step in empirical VAR analysis is to determine the lag length in the underlying VAR. A common approach is to compute information criteria (Akaike, Schwarz and Hannan-Quinn) and choose the lag length that minimizes these measures. The workflow should start with a maximum lag length and then we compute these criteria for each lag length $p = 0, \dots, p_{\max}$ using the same number of observations for each lag length. The function `pfind.m` produces the following output assuming that $p_{\max} = 12$.

p	SIC	HQC	AIC
0	7.7402	7.7252	7.7155
1	-7.9446	-8.0047	-8.0434
2	-8.2387	-8.3438	-8.4115
3	-8.1466	-8.2968	-8.3935
4	-8.0537	-8.2489	-8.3746
5	-7.9827	-8.223	-8.3777
6	-7.8802	-8.1654	-8.3492
7	-7.7906	-8.1209	-8.3337
8	-7.6997	-8.075	-8.3168
9	-7.6104	-8.0307	-8.3016
10	-7.5102	-7.9756	-8.2755
11	-7.4194	-7.9298	-8.2587
12	-7.318	-7.8735	-8.2314

The minimum values of all three criteria suggest that the lag length $p = 2$ in the underlying VAR.

Alternatives to using information criteria as outlined above is to use one of the following approaches:

- Top-down sequential testing (general-to-specific): The VAR(p) model is

$$y_t = \nu + A_1 y_{t-1} + A_2 y_{t-2} + \dots + A_p y_{t-p} + u_t$$

where $\nu = A_0$. Start with a maximum number of lags p_{\max} testing a sequence of null hypotheses: $\mathbb{H}_0: A_{p_{\max}} = 0$ vs. $\mathbb{H}_1: A_{p_{\max}} \neq 0$, $\mathbb{H}_0: A_{p_{\max}-1} = 0$ vs. $\mathbb{H}_1: A_{p_{\max}-1} \neq 0$, ..., $\mathbb{H}_0: A_1 = 0$ vs. $\mathbb{H}_1: A_1 \neq 0$. Process terminates when there is a rejection. Use Wald or LR tests.

- Bottom-up sequential testing (specific-to-general): Reverse the procedure above, start with p_{\min} testing for autocorrelation in the residuals (using for example a multivariate test). Add lags until there is no significant autocorrelation.

Answers using either of these two approaches should also be accepted and receive full points if correctly implemented and explained.

3. Test for multivariate autocorrelation, heteroscedasticity and normality. Does your model satisfy the underlying assumptions?

Answer: To verify that the model is well-specified, we next test for autocorrelation and heteroscedasticity in the residuals and the null hypothesis that the residuals are normally distributed. We should use multivariate tests provided in the functions: `portman.m`, `march.m` and `multnorm.m`. First we need to re-estimate the VAR(2) model using `VARls.m`. We allow for a constant term but no linear trend. The argument used to exclude a linear trend is that the VAR in levels can be re-written as a VAR in first differences (and as a VEC model) by subtracting y_{t-1} from both sides of the levels VAR leaving the constant term and the residuals unaffected.

Start with the multivariate test for autocorrelation. We need to specify the horizon used when computing the test statistic. We start with horizon equal to 3. The `portman.m` function gives us the following result:

Tested order:	3
Test statistic	17.61
p-value	0.039982
Adjusted test statistic	17.69
p-value	0.038949
degrees of freedom	9

Here we note that the null hypothesis is rejected at the 5 percent level. It seems that there is significant autocorrelation in the residuals. There are two ways to proceed. Either adding a lag to the underlying VAR or check whether the problem with autocorrelation persists for other horizons. We start by testing for autocorrelation assuming that $p = 2$ but allowing the horizon $= 3, \dots, 12$. These tests suggest that we reject the null for horizons 3 and 4 but cannot reject the null of no autocorrelation for longer horizons. The p-values are also very large for these horizons. As an alternative we consider adding one lag to the underlying VAR model. The autocorrelation tests now suggest that the problem with autocorrelation is more pronounced than for $p = 2$, the p-values are consistently smaller. We therefore conclude that the choice suggested by the information criteria is appropriate. A good answer provides a discussion along these lines and present test results for different horizons and for different lag lengths.

Next, we test for multivariate ARCH using the `march.m` function. The results suggest that we, for horizons $= 3, \dots, 12$ always reject the null hypothesis that there are no ARCH effects in the residuals at the 5 percent level. For horizon $= 3$ we obtain:

Test	Doornik-Hendry
test statistic:	106.3
p-value	0.0053287
degrees of freedom	72

Adding one lag to the underlying VAR model does not change this result. To investigate this potential problem further we turn to univariate ARCH tests, the Engle's ARCH test. These tests suggest that there is no ARCH effect in any of the three residuals, thus contradicting the multivariate tests, irrespective of the horizon. It is very often the case that the multivariate ARCH test tends to reject the null even when there is no ARCH effect in the residuals is discussed in the curriculum. A good answer provides a discussion and should end with Engle's ARCH test for different horizons showing that there is no ARCH effect in any of the residuals irrespective of the horizon.

Finally, we test the null that the residuals are normally distributed using the `multnorm.m` function. We obtain the following results:

Test	Doornik-Hansen	Lütkepohl
joint test statistic:	9.8051	9.2477
p-value	0.1331	0.16012
degrees of freedom	6	6
Skewness only	0.090103	0.13371
p-value	0.993	0.98751
kurtosis only	9.715	9.114
p-value	0.02115	0.027813

The joint test statistic suggests a non-rejection of the null that the residuals are normally distributed. Individual tests suggests the presence of kurtosis but no skewness at the 5 percent level.

Overall these test statistics suggest that the model with $p = 2$ is fairly well-specified. Adding one lag does not solve any problems detected by the multivariate autocorrelation and ARCH tests, on the contrary, the problems are intensified. Furthermore, univariate ARCH tests strongly contradict the multivariate test. A good answer provide a discussion and should end with the conclusion that $p = 2$ seems reasonable.

Testing for cointegration

4. For your preferred model, proceed by testing for cointegration using the MATLAB function `jcitest`. Explain your approach and how you proceed to

find the number of cointegration vectors in the system, that is the rank r . Do you use different sources of information when determining the rank? If so, explain how you argue.

Answer: Assuming that $p = 2$ in the underlying VAR implies that there is only 1 lag in the VEC model. Remember that the MATLAB function `jcitest` requires the user to specify $p - 1$ instead of p , a good answer must explicitly mention the number of first difference lags used in the tests. Using the `jcitest` function we obtain:

r	h	stat	cValue	pValue	eigVal
0	1	143.0489	29.7976	0.0010	0.2295
1	0	5.8971	15.4948	0.7200	0.0100
2	0	0.5959	3.8415	0.5979	0.0011

Starting with the hypothesis that $r = 0$ we find that the null hypothesis that $r = 0$ is rejected at the 1 percent level. Increasing the rank we find that we cannot reject the null that $r = 1$. The data, therefore, suggest the presence of 1 cointegration vector.

This result holds for any lag length. So if it was decided to increase the number of lags in the underlying VAR above to handle the autocorrelation and ARCH findings, the Johansen test still suggest the presence of 1 cointegration vector.

We can use the estimated eigenvalues as an alternative source of information when determining the rank. As is clearly illustrated in the table above, the eigenvalue falls substantially when increasing the rank. Our conclusion that the rank is equal to 1 is clearly supported.

A good answer must include a description of the approach, the so called Pantula principle, where we start by considering the null hypothesis that the rank is 0 and then we test the null that the rank is 1 and so on. A sensitivity analysis of the unimportance of the number of lags should also be included. As mentioned above, the answer must explicitly state how many lags there are in the VEC model estimation.

5. Discuss and argue for the preferred way to include deterministic components in the model.

Answer: The Johansen test computed above rests on the assumption that there is no linear trend in the cointegration vector. We can distinguish between five different cases depending on assumptions about deterministic components. In our case, it is evident that we need a constant term in the VEC model. The question is whether we also should add a linear trend, either a linear trend in the cointegration vector but no quadratic trend in the data or a allowing for a quadratic trend in the data.

We can test the null hypothesis that there is a linear trend in the cointegration vector using the `jctest` function in MATLAB. The MATLAB code to be used is:

```
[h,pValue] = jctest(y,1,'Bcon',[0 0 0 1]','model ','H*','lags',p-1);
```

Executing this command we find a test statistic of 1.8357 with a p-value of 0.1755, thus indicating a non-rejection of the null hypothesis that the linear trend can be excluded from the cointegration space.

A good answer should include a formal test of this null hypothesis and a motivation based on the graph of the real exchange rate where it was clear that there is no linear trend in the PPP relation, i.e., no linear trend in the real exchange rate.

6. Impose your preferred rank and test hypotheses on the cointegration space using the MATLAB function `jctest`. Start with tests for exclusion, stationarity and weak exogeneity. Explain the meanings of these tests.

Answer: Stationarity: the null hypothesis that a specific variable is stationary. Weak exogeneity: the null hypothesis that a specific variable respond to deviations from the long-run relationship (the cointegration vector). Exclusion: the null hypothesis that a specific variable can be excluded from the cointegration space.

We can test all these hypotheses using the MATLAB function `jctest`. The results suggest, overwhelmingly, that we can reject the null that the variables are stationary, that any variable can be excluded from the cointegration space and that the variables do not respond to deviations from the cointegration vector. All these results must be shown in a table (or printout from MATLAB).

7. Test the null hypothesis that PPP holds using the MATLAB function `jctest`. Explain how this test relates to the exclusion and stationarity tests.

Answer: The MATLAB code to conduct this test is:

```
[h,pValue] = jctest(y,1,'Bvec',[1 -1 -1]','lags',p-1);
```

Depending on data sets, we obtain different results here. In some cases we strongly reject the null that PPP holds (for this data set we reject the null at the 1 percent level) but for other data sets we cannot reject the null. The p-values from these tests must be provided.

Stationarity test, exclusion tests and tests for specific cointegration vectors are related. If one variable can be excluded from the cointegration space, then we expect to reject cointegration vectors where this variable is included. If a variable is stationary, then this variable forms a cointegration vector where all other variables are excluded. The test results should be consistent, but this may not always be the case in practice. In our empirical application we found that we reject the null hypothesis that the variables

are stationary and we also rejected the null that any variable can be excluded from the cointegration space. Then we tested for PPP and found that this null can be rejected. This implies that we reject PPP but we have not rejected the hypothesis that the three variables form a cointegration vector. The estimated cointegration vector is $[1 \ -1.05 \ -2.40]'$ under the assumption that the rank is 1. This vector is significantly different from the PPP relation.

8. Split the sample into two equal sized sub-samples and perform tests for exclusion, stationarity, weak exogeneity and the null hypothesis that PPP holds. Comment on the importance of the sample length for these tests.

Answer: Here we will find that the information criteria still suggest that $p = 2$, Johansen test often are consistent, that is, suggest rank equal to 1, but tests for stationarity, exclusion and weak exogeneity do change substantially. For example, using our data we now find that nominal exchange rate is stationary, the domestic price level does not adjust to deviations from the cointegration vector and that the foreign price level and the nominal exchange rate can be excluded from the cointegration space using the first part of the sample. For the second part of the sample we find that we always reject stationarity, always reject weak exogeneity but reject exclusion only for one variable.

We know that the Johansen test and tests on the cointegration vectors in particular are biased in small samples. For our sample sizes (and of course the simulated data) we only find that tests on the cointegration vector change when reducing the number of observations.

9. Impose $r = 1$ and the theoretical cointegration vector (the assumption that PPP holds) and re-estimate the VEC model using the full sample and using your preferred lag length found above. Comment on the driving forces of the data in this model.

Answer: We can estimate the VEC model with known cointegration vector using either LS or ML. In our model we have rank equal to 1 and 3 variables. This implies that there are 2 common trends driving the 3 variables. These two common trends determine the long-run behavior of the three time series, that is, they have long-run effects on all variables. But, given our assumption that PPP holds, the long-run effects on the real exchange rate must be equal to zero. Since we have one cointegration vector, we have 1 transitory shock affecting the variables only in the short-run.

A good answer includes both estimates of the VEC model and a short discussion about the driving forces as above.

Identification of structural model

10. What are the main differences between short- and long-run restrictions. What are the main assumptions underlying these two alternatives to identify structural VAR models? You are not required to discuss over-identification or non-recursive identification.

Answer: The main difference between short- and long-run restrictions is that the former imposes restrictions on the contemporaneous effects of the shocks (the first period effect) whereas the latter imposes restrictions on the long-term effects of the shock. These restrictions can take on many different forms, they could be exclusion restrictions, proportionality restrictions or equality restrictions.

Short-run restrictions: A common approach to implement these restrictions is to use a Cholesky decomposition of the variance-covariance matrix of the residuals. Consider the VAR(p) model

$$y_t = \nu + A_1 y_{t-1} + A_2 y_{t-2} + \dots + A_p y_{t-p} + u_t$$

where $\nu = A_0$ and $E[u_t u_t'] = \Sigma_u$. This is the reduced form VAR(p) model. A structural form VAR(p) model can be written as

$$B_0 y_t = B_1 y_{t-1} + \dots + B_p y_{t-p} + w_t$$

or as

$$y_t = \underbrace{B_0^{-1} B_1}_{A_1} y_{t-1} + \dots + \underbrace{B_0^{-1} B_p}_{A_p} y_{t-p} + \underbrace{B_0^{-1} w_t}_{u_t}$$

where $E(u_t u_t') \equiv \Sigma_u = B_0^{-1} (B_0^{-1})'$ and $E(w_t w_t') = I_K$. We can estimate the reduced form VAR to obtain estimates of A_1, \dots, A_p and Σ_u but identification requires a mapping from these estimates to the parameters B_0, B_1, \dots, B_p . A simple and popular approach is to use a Cholesky decomposition (or orthogonalization) to identify the parameters in the structural VAR model. Define a lower-triangular $K \times K$ matrix P with positive diagonal such that $\Sigma_u = P P'$. Since $\Sigma_u = B_0^{-1} (B_0^{-1})'$ we then find that $B_0^{-1} = P$.

The reduced form VMA model is

$$y_t = \delta + \Phi(L) u_y$$

where $\Phi(L) = A(L)^{-1}$ and L is a lag operator. The structural form VMA model is

$$y_t = \delta + \Phi(L) u_y = \delta + \Phi(L) B_0^{-1} (B_0^{-1})' u_t = \delta + \Theta(L) w_t$$

Note: The Cholesky decomposition allows us to just identify the structural parameters, we need $K(K-1)/2$ restrictions to just identify the structural model and the Cholesky decomposition provides us with those restrictions

since P is lower (or upper) triangular. Moreover, the structural model is recursive, i.e., we impose a causal chain implying that the ordering of the K variables matter. We obtain different P matrices for different ordering of the variables.

Long-run restrictions: If at least one of the variables included in the VAR model contains a unit root we can impose restrictions on the long-run effect of a structural shock. In particular, this approach allows us to impose economic restrictions that are not controversial (in contrast to short-run restrictions are), for example that nominal shocks are neutral in the long-run (the classical nominal-real dichotomy). One example is the Blanchard-Quah identification within a bivariate VAR model with output and unemployment where it is assumed that there are two shocks, a demand and a supply shock where output is neutral with respect to the demand shock in the long-run. This identification approach can be extended to allow for additional shocks. Unit roots and cointegration relationships allow us to consider the identification of permanent shocks (shocks having long-run effects on at least one variable in the system) and the identification of transitory shocks (shocks having only short-run effects) on the system. In the Blanchard-Quah identification we assume that output is a unit root process whereas unemployment is stationary (and thus constitute a cointegration vector). The advantage of using this approach is that there is a closer relationship between economic models and empirical analysis where restrictions (including possible cointegration vectors) can aid the identification.

To illustrate this approach, consider the following VEC model

$$\Delta y_t = \alpha \beta' y_{t-1} + \Gamma_1 \Delta y_{t-1} + \dots + \Gamma_{p-1} \Delta y_{t-p+1} + u_t$$

where α is the adjustment coefficients and β is the cointegration vector. The VEC model can be rewritten as a Common Trends model

$$y_t = \Xi \sum_{i=1}^t u_i + \Xi^*(L)u_t + y_0^*$$

where

$$\Xi = \beta_{\perp} \left[\alpha'_{\perp} \left(I_K - \sum_{i=1}^{p-1} \Gamma_i \right) \beta_{\perp} \right]^{-1} \alpha'_{\perp}$$

where α_{\perp} and β_{\perp} are orthogonal complements to α and β respectively. The structural Common Trends model can be written as

$$y_t = \Xi \sum_{i=1}^t B_0^{-1} w_i + \Xi^*(L)B_0^{-1}w_t + y_0^*$$

$$y_t = \underbrace{\Xi B_0^{-1}}_{\Upsilon} \sum_{i=1}^t w_i + \Xi^*(L)B_0^{-1}w_t + y_0^* = \Upsilon \sum_{i=1}^t w_i + \Xi^*(L)B_0^{-1}w_t + y_0^*$$

where Υ is the matrix of long-run multipliers, it measures the long-run effect of the common trends (or the permanent shocks). Note that the long-run effects of the stationary part $\Xi^*(L)B_0^{-1}w_t$ goes to zero as $j \rightarrow \infty$. The rank of Υ is the same as the rank of Ξ , i.e., rank $K - r$. Long-run restrictions can be imposed directly on Υ , if the long-run effect of a shock is zero on all variables, then the corresponding column of Υ is restricted to zero.

To identify the transitory shocks we impose restrictions on B_0^{-1} , in particular, we impose restrictions on the last r columns of this matrix.

11. Write down the reduced form and structural form Common Trends model consistent with the VEC model. Show how these two representations are related. Provide an interpretation of the three structural shocks in the structural model. If you cannot provide names for these shocks, try to explain how they affect the data and the real exchange rate under the maintained assumptions.

Answer: The reduced and structural forms are provided above. We start with the permanent shocks. In a standard AD-AS model we know that demand shocks have short-run effects on inflation, inflation should either increase or decrease temporarily. During this process, the price level must adjust to either a higher or a lower level. In our trivariate system with domestic and foreign price levels we may assume that the two permanent shocks are associated with domestic and foreign demand shocks, they have permanent effects on the price level within their own economy but inflation is unaffected in the long-run. If we also assume a small open economy, we may add the assumption that demand shocks in the small open economy should not affect prices in the long-run. This assumption can be used to separate the two permanent shocks, i.e., to identify these two shocks. Since PPP holds, these two shocks only have short-run effects on the real exchange rate.

The transitory shock could be viewed as an exchange rate shock leading to transitory fluctuations in the nominal exchange rate. It is implicit in the PPP model that any change in the relative price level leads to adjustments in the nominal exchange rate, not the other way around.

A good answer must include a discussion of possible shocks affecting the VAR/VEC system. The arguments above may not be the only available option, but it is essential that the answer includes a motivation and main arguments based on economic model or intuition.

12. What is the consequence for the long-run multiplier if we assume that PPP holds?

Answer: The long-run multiplier Υ is a 3×3 matrix where the third column is zero since the long-run effects of the transitory shock on all variables is zero. However, the condition that $\beta'\Upsilon = 0$ is required for the identification

of the permanent shocks. The assumption that PPP holds therefore implies that the long-run effect on the real exchange rate is equal to zero.

13. Use the estimated reduced form VEC model together with identifying restrictions (a combination of short- and long-run restrictions) to identify the three structural shocks. Show how these shocks can be identified including the number of long-run and short-run restrictions needed to just identify the structural shocks. Consider only null restrictions.

Answer: The small open economy restriction mentioned above implies that the first element in the second row of Υ is zero. This assumption is enough to identify the two permanent shocks. The transitory shock is identified since the cointegration vector is orthogonal to the common trends.

14. Outline how the MATLAB solver can be used to impose these restrictions.

Answer: The following code implements the restriction:

```
% restrictions.M
% Normalization: SIGMA_w=I
function q=restrictions(B0inv)
global GAMMA SIGMA alpha beta alpha_perp beta_perp Xi p
K=size(B0inv,1);
THETA1=Xi*B0inv;
F=vec(B0inv*B0inv'-SIGMA(1:K,1:K));
% Long run and short run restrictions
q=[F; THETA1(2,1); THETA1(1,3); THETA1(2,3); THETA1(3,3)];
q'+1;
```

Impulse responses and forecast error variances

15. Use the MATLAB solver to identify the structural model. Check that the solver provides a valid identification and compute the variance-covariance matrix of the identified structural shocks. Please, provide the MATLAB code you are using to identify the shocks in the appendix. It must include a description of the null restrictions you impose.

If you fail computing the B_0^{-1} matrix above, please use the restrictions.p file. This file works as a standard m-file but the coding is concealed and there is no way to convert the p-file into an m-file. Note that the restrictions.p file is set up to use the MATLAB solver to compute the B_0^{-1} matrix using a generic identification based on estimates from the VEC model. You can use the standard coding to initiate the solver. To do this, you need to include the following code into your MATLAB m-file. Note: Make sure that you don't have any restrictions.m files in the same folder and that

the function `vec.m` is in the same folder. The same file can be used in a bootstrap.

```
% Set global variables
global SIGMA GAMMA alpha beta alpha_perp beta_perp Xi p;
% Set options for fsolve
warning off
options=optimset('TolX',1e-10,'TolFun',1e-10,'MaxFunEvals',1e+10,'MaxIter',2000);
% Compute B0inv
B0inv=fsolve('restrictions',randn(K,K),options);
```

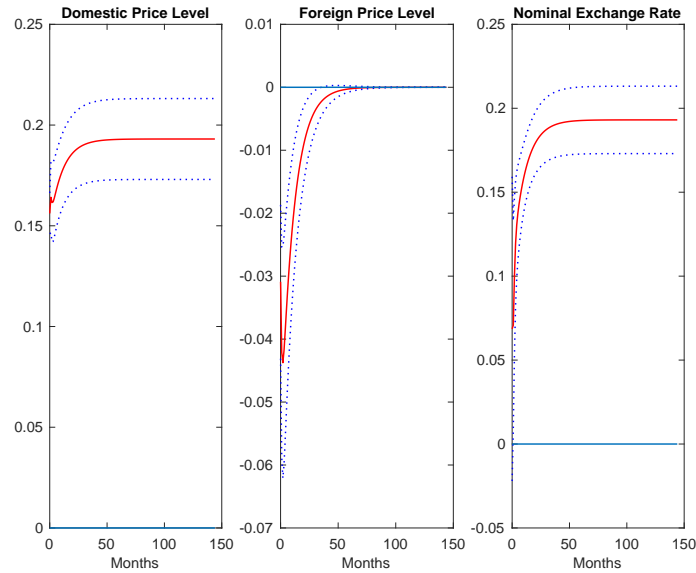
Answer: The code given above can be used to estimate the structural model. Note that the `restrictions.p` file implements another identification scheme, it may not be possible to provide an interpretation of the two permanent shocks. However, using this generic identification allows the user to estimate a structural model. In either case, it is necessary to show that the solver has found a proper solution. Apart from providing an estimate of the B_0^{-1} matrix, it is necessary to show the following matrices: $B_0^{-1}(B_0^{-1})' - \Sigma_u$ which should be equal to zero; Υ which should include the zero restriction imposed on one element in the first two columns; $B_0 \Sigma_u B_0'$ which is the variance-covariance matrix of the structural shocks w_t which should be an identity matrix.

A good answer implements the proposed identification stated in question 11 correctly and should also provide statistics verifying that the solution is valid using the statistics mentioned in the previous paragraph.

16. Estimate the structural VAR model and compute impulse response functions (with bootstrap confidence bands using the delta method, i.e., the standard residual based recursive design bootstrap with intervals based on bootstrap standard errors) and variance decompositions (with bootstrap standard errors using Efron's percentile intervals). In addition, compute the implied impulse responses of the real exchange rate to all shocks and the corresponding forecast error variance decomposition. You can show forecast error variance decompositions in either a table or in a graph. Interpret your results.

Answer: See graphs below. As is evident from the IRFs, we have imposed the restriction that the long-run effect from domestic demand shocks (domestic price shock) on the foreign price level is zero. A good answer includes both the graphs of the IRFs and the FEVDs and a discussion of the results. Answers excluding the effects of the three shocks on the real exchange rate are enough. A very good answer includes these graphs. To compute the FEVDs requires some additional coding in MATLAB, there is no preprogrammed function to do this. However, it is straightforward to rewrite the function `FEVD.m` to compute the variance decompositions based on IRFs of the real exchange rate. To compute the IRFs of the real exchange rate we simply multiply the responses of the three variables to,

Figure 3: Effects of a domestic demand shock. 95 percent confidence bands are computed using the delta method with 500 replications.



say, the first shock with the PPP relation. The delta method produces symmetric confidence bands around the point estimates whereas Efron's percentile approach is asymmetric taking into account that the variance decomposition cannot be negative or exceed 1 (or 100%). Confidence bands violating this latter condition must be incorrect.

A very good answer should also include a statement about whether or not the signs of the columns of the B_0^{-1} matrix is handled. It is necessary to check that the signs of the columns are consistent for each replication. In practice this may not be a problem in the present setting, but should be mentioned. Furthermore, a statement about the convergence of the solver in each replication should be added.

17. Show a plot of the accumulated permanent shock(s) and the three variables. Discuss the results. Then plot the real exchange rate together with the accumulated permanent shock(s). Is it possible to draw conclusions regarding the driving forces of the variables?

Answer: There are different ways to compute the common trends in the estimated model. One idea is to simply compute the accumulated sum of the two permanent shocks, another idea is to multiply the accumulated permanent shocks by the Ξ matrix defined above. Either way, we need to remove the deterministic components of the data (the two prices). As an illustration we compute the two common trends using the latter approach and plot these two components together with the data where we have removed a linear trend from the two price levels and the real exchange rate.

Figure 4: Effects of a foreign demand shock. 95 percent confidence bands are computed using the delta method with 500 replications.

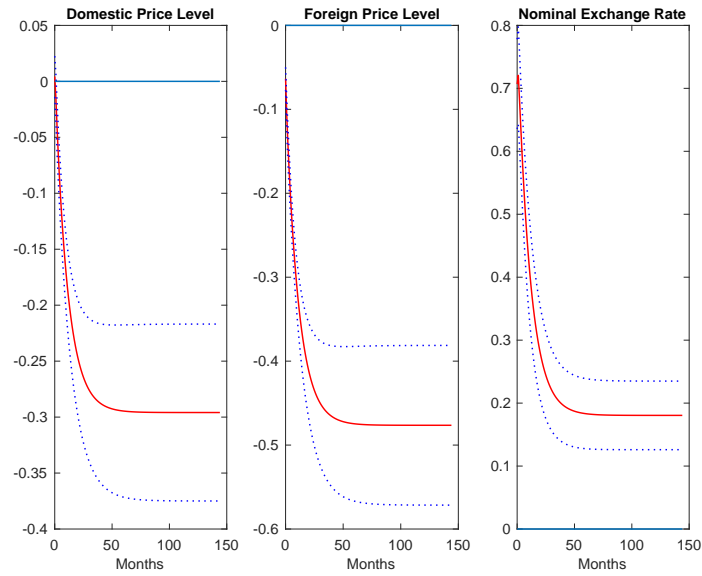


Figure 5: Effects of all three shocks on the real exchange rate. 95 percent confidence bands are computed using the delta method with 500 replications.

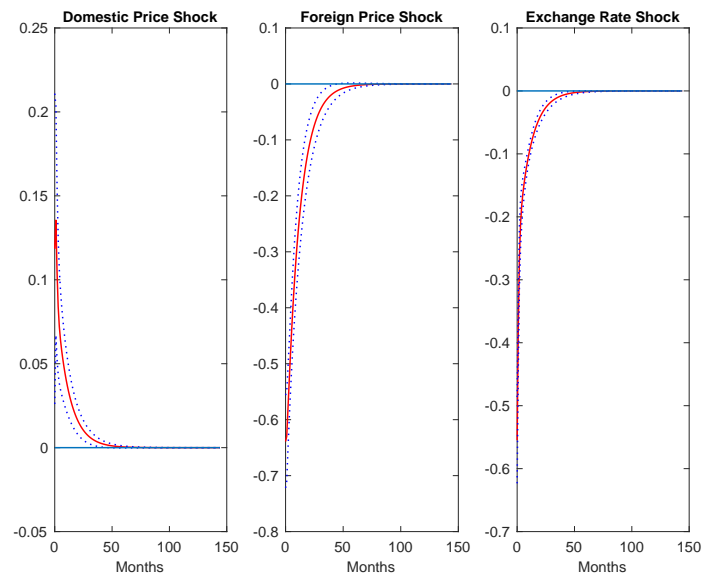


Figure 6: Forecast error variance decomposition, domestic demand shock. 95 percent confidence bands are computed using Efron's percentile method with 500 replications.

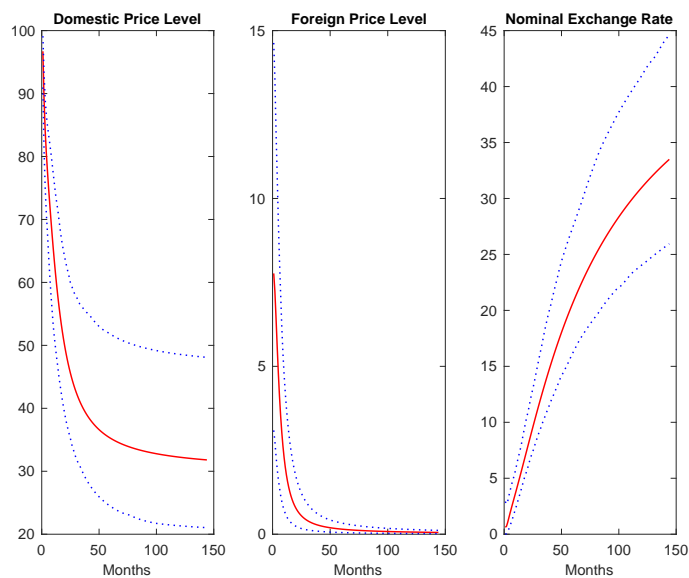


Figure 7: Forecast error variance decomposition, foreign demand shock. 95 percent confidence bands are computed using Efron's percentile method with 500 replications.

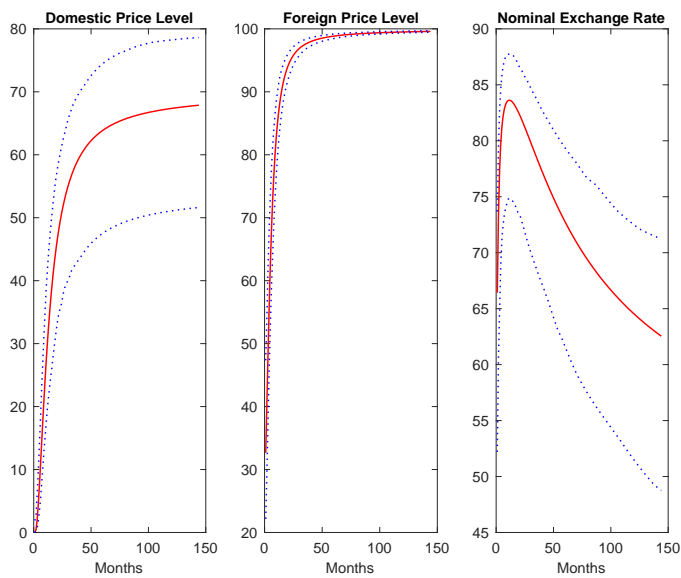
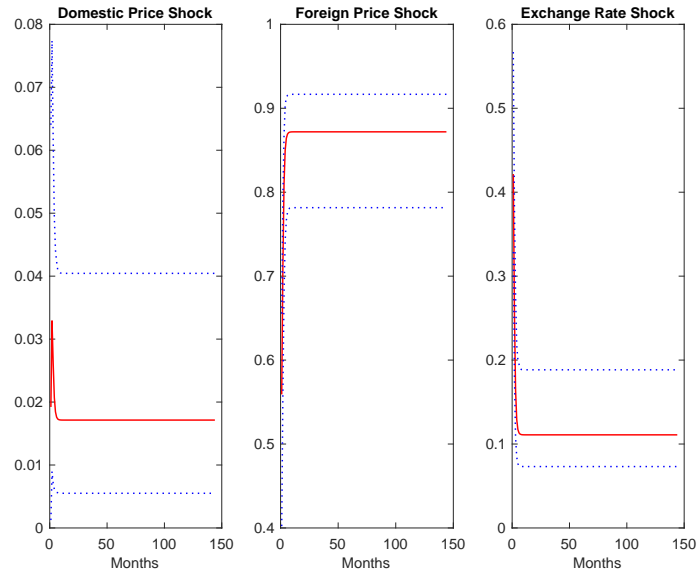
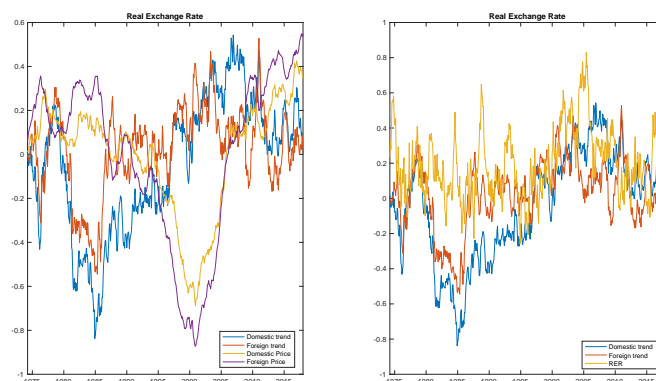


Figure 8: Forecast error variance decomposition, real exchange rate. 95 percent confidence bands are computed using the delta method with 500 replications.



The two common trends seem to be related to the two price levels (the detrended price levels), the correlation is negative so that an increase in the common trend is associated with a fall in the price level. The real exchange rate is only associated with fluctuations in the common trends during the latter part of the sample. Together, this suggests that our identification of the two permanent shocks and their hypothesized relationship to the price levels cannot be rejected.

Figure 9: Estimated common trends and the data.



Extensions

18. Re-estimate the VEC model using the estimated cointegration vector instead of the theoretical one. Use the same number of lags as above and compute impulse response functions (with bootstrap confidence bands using the delta method). Compare these to the impulse response functions you obtained when imposing the PPP condition. How sensitive are impulse responses to deviations from PPP? Compute the impulse responses of the real exchange rate. Interpret and compare to the case when PPP holds. As above, in case you have not managed to use the solver, you can use the p-file restrictions.p to perform these computations.

Answer: The only difference is that the long-run effects on the real exchange rate are non-zero. Otherwise the impulse response functions should look the same. A good answer includes a discussion of this fact and an explanation, the explanation being that PPP does not hold and therefore the impulse responses of the real exchange rate do not converge to zero.

19. An alternative to using the MATLAB solver to compute the B_0^{-1} matrix is to use the approach suggested by Warne (1993). Outline this approach and show that the Warne approach yields a B_0^{-1} matrix identical to the one found by the solver. You can either work with the theoretical cointegration vector assuming that PPP holds or the estimated cointegration vector. Please, provide the code you are using in the appendix.

Answer: It is enough here to show that the B_0^{-1} matrix obtained using the Warne approach is equal to the solution obtained using the MATLAB solver. The answer must include the code used to compute the closed form solution.

Following Warne we first define the matrix Υ_0 to identify the the two permanent shocks and then we define the U matrix. First of all we need to re-order the time series vector. We put the foreign price level first and then follows the domestic price level and the nominal exchange rate. The reason for this is that it will be simple to find an Υ_0 matrix leading us to the Υ matrix where we have imposed one zero restriction, the restriction that the domestic demand shock cannot affect the foreign price level in the long-run. The cointegration vector, under the assumption that PPP holds, will then be $[-1 \ 1 \ -1]$. From Warne we know that $\beta'\Upsilon_0 = 0$ and that $\Upsilon = \Upsilon_0\pi$ where π is a lower triangular matrix. Let

$$\Upsilon_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}.$$

It is easily seen that $\beta' \Upsilon_0 = 0$. The matrix Υ is then given by

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \pi_{11} & 0 \\ \pi_{21} & \pi_{22} \end{bmatrix} = \begin{bmatrix} \pi_{11} & 0 \\ \pi_{21} & \pi_{22} \\ -\pi_{11} + \pi_{21} & \pi_{22} \end{bmatrix}$$

where we see that the identifying restriction that the domestic price shock has no long-run effect on the foreign price level (remember that we have reordered the shocks).

Since we have only 1 transitory shock, $U = [0 \ 0 \ 1]$.

It is now straightforward to code this. For this ordering of the variables we can verify that the B_0^{-1} matrix found using the Warne method is equal to the one found by the solver. It could be that we need to switch signs on one or two columns of the solver solution, but the absolute values are identical. The code used could look like (the code is adapted from the KLversusWarne.m file). The answer must include the MATLAB code and the definition of the matrices above.

```
% Warne identification
beta=[-1 1 -1]';
Upsilon0 = [1 0 -1;0 1 1]';
MHLPI=inv(Upsilon0'*Upsilon0)*Upsilon0'*Xi;
pipit=MHLPI*SIGMA*MHLPI';
pimat=chol(pipit)';
Upsilon=Upsilon0*pimat;
Fk=inv(Upsilon'*Upsilon)*Upsilon'*Xi;
display(Fk,'Fk matrix');
Umat=zeros(1,3);
i=1;
while i<=r;
Umat(i,K-i+1)=1;
i=i+1;
end;
% Check that identification of transitory shocks is valid
if det(Umat*alpha)==0
display('Identification of transitory shock is invalid');
else
display('Identification of transitory shocks is valid');
end
xi=alpha*inv(Umat*alpha);
i=1;
while i<=K;
j=1;
while j<=r;
if abs(xi(i,j))<=1E-12;
xi(i,j)=0;
```

```

else
end
j=j+1;
end
i=i+1;
end
qr=chol(xi'*inv(SIGMA)*xi)';
Fr=inv(qr)*xi'*inv(SIGMA);
display(Fr,'Fr matrix');
invB0 = inv([Fk;Fr]);
invB0 = [-invB0(:,1) invB0(:,2:3)];
display(invB0,'B0^-1 matrix');
display(-Xi*invB0,'(3) C(1)*B0^-1 should be Upsilon zeros(K,r)');
display(inv(invB0)*SIGMA*inv(invB0)'),'(4) Covariance matrix of structural
shocks w_t should be I_K');
display(inv(qr)*xi'*inv(SIGMA)*xi*inv(qr)'),'Should be diagonal');

```